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Application of Riemann Problem Solvers to Wave Machine Design

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Nomenclature

p	= pressure, N/m^2
ρ	= density, kg/m^3
u	= velocity, m/s
x	= space coordinate (along rotor passage)
t	= time coordinate
H. P.	= high pressure
L. P.	= low pressure
[L]	= left
[R]	= right

Subscripts

1, 2, 3 ..., 10 = states shown in Fig. 1

The terms tubes, passages, and cells have been used interchangeably in the text.

Introduction

WAVE rotors are devices in which wave propagation is used to effect a transfer of energy between a gas and a rotating shaft or directly between one gas and another. Reviews of such devices and their general operating principles have been published[1,2] and their commercial applications have been developed.[3]

In the case of direct transfer of energy, one gas (driver) at high pressure is used to compress a second gas (driven). The process is arranged to occur in tube-like passages arranged on the periphery of a drum or rotor. The compression is achieved by means of compression waves or shock waves and the compressed gas is drawn off from the end of the tube in which the process takes place. The driver gas then undergoes a series of expansions to a lower pressure and is scavenged out by freshly inducted driven gas at approximately the same pressure level. This fresh "charge" is then compressed by the high pressure driver gas and the cycle repeats itself.

Steady rotation of the drum sequences the passage of the ends of the tubes past stationary inlet ports, outlet ports, and endwalls. This establishes unsteady but repetitive flow processes within the rotating tubes and essentially steady flow in the inlet and outlet ports.

Complicated unsteady wave phenomena appear in the working of these devices, and design of even the most simple mode of operation requires calculation of an array of wave processes such as shock waves, reflected and "hammer" shock waves (from a wall or moving interface), rarefaction waves (connecting two states of a gas or produced by tube end closure), as well as interactions between incident and reflected waves. In general, the two gases in these devices have considerably different ethalpies, leading to the formation of contact surfaces that also interact with the various waves. The flow in each tube is usually analyzed using one-dimensional or quasi-one-dimensional approaches, implying that changes in state occur only along the passage. This enables the wave processes to be depicted on an x-t plane. Such wave diagrams are extremely useful in the examination of possible wave cycles, providing visual information of the wave paths, proper placing of the ports, seeing interactions, and whether a particular cycle, "closes," the latter being necessary for cyclic operation. The construction of wave diagrams is usually quite involved, requiring considerable time and effort, since a mismatch in any one parameter gives rise to a new pressure wave which has to be accounted for through the complete rotational cycle. The method of characteristics, despite its limitations (only weak shocks are permitted), has been the technique generally used for carrying out cycle computations.[1] However, graphical methods of constructing wave diagrams are time consuming and may require weeks of tedious calculations.[4] Clearly, a fast and "user friendly" computational procedure is required to carry out preliminary design calculations for wave rotor devices.

The method described below offers a unified approach to the calculation of wave rotor cycles with no restriction on the strength of the waves involved.

The method has been exercised in the design of a wave-turbine experiment at the Naval Postgraduate School's

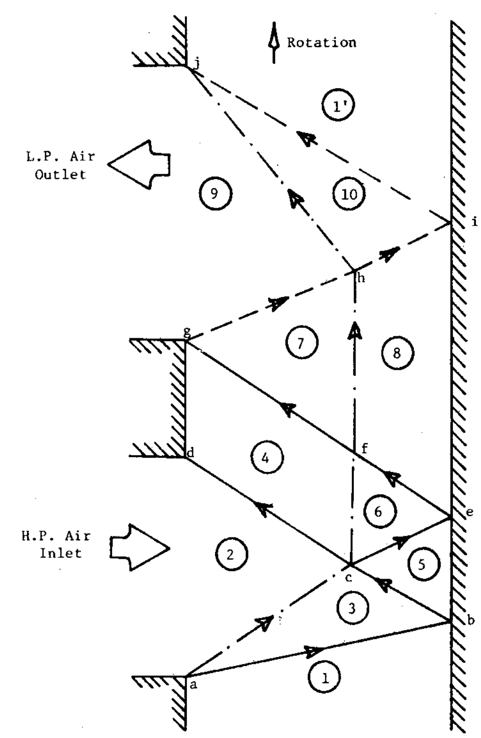

Fig. 1 Simplified wave diagram for "impulse turbine mode" operation.

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Turbopropulsion Laboratory. The work is part of a program to examine wave rotors and their potential as components in flight propulsion engines.

Method

The method uses an approach described in Ref. 5 to solve the general "Riemann problem"—that of finding the flow which results when two gases, each at some specified initial state, are suddenly brought into contact with each other. The initial states are completely defined by specifying the pressure density and velocity of each gas. Depending on the initial data, four types of discontinuity propagations are possible, involving combinations of shock and rarefaction waves originating from the point of initial contact and proceeding to the left or to the right. The solution defining the final state is analytical and does not impose restrictions on the initial parameters of the gases. The type of discontinuity which results is defined by the solution and is not prescribed beforehand.

A computer code that solves the Riemann problem for any given initial states of the two gases has been developed. (The program in PASCAL or FORTRAN is available from the authors upon request.) The program is set up such that the gas with the higher initial pressure may be on either side. The results give the type of discontinuity, e.g., shock-shock, shock-rarefaction, etc., the pressure and velocity at the interface and the densities on either side of the interface. Velocities of propagation of the waves involved are also computed, with two velocities calculated for rarefaction fans corresponding to the head and tail waves. The pressure at the interface is computed by an iterative procedure, using the average of the two initial pressures as the initial value. Entropy checks are incorporated into the program which disallow the appearance of expansion shocks. Although the program, in its present form, can handle only gases which have the same ratio of specific heats, it can quite readily be modified to deal with different specific heat ratios, as is outlined in Ref. 6. The proposed wave-turbine and subsequent experiments involve only relatively cold air without combustion products, and hence cycle calculations with different specific heat ratios have not been necessary.

The following section describes the wave-turbine as an example of how cycle calculations may be carried out using the program.

Example

Figure 1 shows the wave diagram (not to scale) for the proposed "turbine mode" experiment in which a wave rotor would be used to work as an impulse turbine and produce shaft power output. For clarity, the waves have been shown as single, straight lines, with full lines indicating compression or shock waves, chain-dotted lines indicating contact discontinuities, and dashed lines representing expansion fans. The direction of rotation of the rotor is as indicated by the arrow at the bottom of the figure. The right side of the rotor is blocked off, and the left side tube ends open to inlet and outlet ports alternately. The encircled numbers depict regions of uniform flow but at different states with respect to adjoining regions. Subscripted flow parameters used in the following discussion correspond to these states. Starting the cycle at state 1 at the bottom of the figure, the rotor tubes are filled with air in a quiescent state and approximately ambient pressure. The cells are then brought into contact with incoming high pressure air at the inlet port. This generates a shock wave (a-b) which propagates into the air at state 1, raising its pressure and density to that of state 3. A mass velocity u_3 is also generated behind the shock. An interface (a-c) separates the incoming air and the compressed air, although for this "cold" configuration the two densities are not very different.

Shock (a-b) reflects off the solid boundary at b as (c-b) and intersects the slower moving interface at point c, where part of the shock is transmitted (c-d) and part is reflected (c-e), creating states 4, 5, and 6. The velocity in state 5 is zero and nearly so in states 4 and 6. The inlet port is closed when the transmitted shock (c-d) arrives at the left end. Shock (c-e) gets reflected again at point e as (e-f) and continues on as (f-g); these secondary reflections are of almost zero strength and do not affect the flow significantly.

At point g, the air outlet port is opened and the compressed air in state 7 exits, with an expansion fan propagating in the opposite direction. The interface bends towards the exit and reaches the end of the passage at point j. The arrival of the interface at point j is timed to match the arrival of the reflected expansion wave (i-j), and the outlet port is closed at this moment. The air in the cells is again in a quiescent state and should be at the same pressure and density as that of the original state 1 for the cycle to be repeated.

The passages in the rotor are 'staggered' at 60 deg to the axis of the rotor, and the change in the angular momentum effected by the reversal in direction of the air flow provides a torque and allows the extraction of shaft work.

The calculations using the Riemann solver code are as follows.

Step 1: H. P. air at state 2 hitting L. P. air at state 1. Initial conditions for Riemann problem solver:

$$[L]: p_2, \rho_2, u_2; \quad [R]: p_1, \rho_1, u_1 = 0$$

Discontinuity type: rarefaction-shock

The shock wave propagates to the right into the rotor passage and the rarefaction propagates to the left into the inlet port. A contact surface follows the shock wave into the passage at a slower speed. In the limiting case where no disturbances are to be propagated into the port (for uniform flow conditions), the initial parameters may be varied to obtain the singular condition in which no wave propagates to the left and a shock wave propagates to the right. This can be described as the discontinuity type "no wave-shock." State 3 is then completely defined.

Step 2: The flow in region 3 is confronted by the "wall" boundary condition at the right side and is brought to a halt by the reflected shock (b-c). This situation is analogous to the flow colliding with a flow at the same state but with equal and opposite velocity. Initial conditions:

$$[L]: p_3, \rho_3, u_3; \quad [R]: p_3, \rho_3, -u_3$$

Table 1 Computed values for "impulse turbine" cycle

	Region										
	1	2	3	4	5	6	7	8	9	10	1′
Pressure, $\times 10^{-5}$ N/m^2	0.943968	1.71453	1.71453	2.97194	2.96339	2.97194	2.97189	2.98056	1.71588	17.2121	0.94569
Density, kg/m^3	1.388	2.185	2.138	3.221	3.145	3.151	3.221	3.157	2.176	2.133	1.406
Velocity, m/s	0	136.8	136.8	0	0	70	70	70	−136.4	−137.1	0

State 5 is completely defined.

Step 3: The reflected shock (b-c) intersects the interface at point c. Initial conditions:

$$[\mathrm{L}]: p_2, \rho_2, u_2; \quad [\mathrm{R}]: p_5, \rho_5, u_5 = 0$$

Solution defines states 4 and 6.

Step 4: Reflected shock (c-e) is re-reflected from the wall at point e. Initial conditions:

$$[\mathrm{L}]: p_6, \rho_6, u_6; \quad [\mathrm{R}]: p_6, \rho_6, -u_6$$

Solution defines state 8.

Step 5: Shock (e-f) hits interface at point f. Initial conditions:

$$[\mathrm{L}]: p_6, \rho_6, u_6; \quad [\mathrm{R}]: \mathrm{p}_6, \rho_6, -u_6$$

Solution defines state 7.

Step 6: Air in the rotor cells is released to exhaust (ambient) conditions. Initial conditions:

$$[\mathrm{L}]: p_{\mathrm{amb}}, \rho_{\mathrm{amb}}, u_{\mathrm{amb}} = 0; \quad [\mathrm{R}]: p_7, \rho_7, u_7 = 0$$

Solution defines state 9.

Step 7: Rarefaction wave (g-h) intersects interface at point h. Initial conditions:

$$[\mathrm{L}]: p_9, \rho_9, u_9; \quad [\mathrm{R}]: p_8, \rho_8, u_8 = 0$$

Solution defines states 9 and 10.

Step 8: Rarefaction wave hits the wall at point *i*, and is reflected in like sense. Initial conditions:

$$[\mathrm{L}]: p_{10}, \rho_{10}, -u_{10}; \quad [\mathrm{R}]: p_{10}, \rho_{10}, u_{10}$$

Solution defines state 1.

This should match with the original state 1 for "cycle closure," which is required if the rotor is to operate continuously.

Table 1 gives the values computed for the cycle and with the procedure described above. The pressures are static values and the velocities are referred to the rotor.

Clearly, a mismatch of rotor speed and/or inlet conditions from design point values will cause new waves to be generated which have to be carried through the entire cycle in order to assess their overall effect on the performance. The Riemann program is particularly useful for preliminary design because of its "building block" approach, which allows walking through any wave configuration state by state. Once a viable cycle is established, a detailed flow solver can be applied to incorporate effects of friction, heat transfer and the finite times taken for cell opening and closing.

Conclusions

The calculation procedure in the example above required only minutes to carry out, with minimal requirements for computer time or storage. (On average, a typical "Riemann Step" calculation required 0.02 s of CPU time on an IBM 370-3033AP computer.) The Riemann problem solver code, therefore, gives a fast, efficient, and unified approach to carry out the preliminary design of wave rotor devices with diverse wave structures and pressure ratios.

It is noted that the code may be coded easily on any home computer and does not require external hardware or software libraries.

Acknowledgment

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Classical Normal Modes in Asymmetric Nonconservative Dynamic Systems

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Introduction

THE dynamic behavior of a general linear discrete system can be described by the vector differential equation

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = 0 \tag{1}$$

where M, C, and K are mass, damping, and stiffness matrices, respectively. The usual treatment of these systems assumes that Eq. (1) is symmetric. Although this assumption is justified for passive systems, in many problems of interest in aeronautics, ship vibrations, and active control of large space structures, Eq. (1) cannot be presented in a symmetric form. This motivates the study of the behavior and properties of this class of problems and also an attempt to derive relations similar to those of symmetric systems.

It is well known that a passive conservative system (i.e., $C=0$, $M=M^T$, and $K=K^T$) possesses normal modes. Rayleigh[1] has shown that a passive nonconservative system possesses normal modes if the damping matrix is proportional to the mass and stiffness matrices, i.e., $C=\alpha M+\beta K$. Furthermore, it has been shown by Caughey and O'Kelly[2-4] that a damped linear symmetric system possesses normal modes if

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